Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 2

15 May 2026

Zone A morning | **Zone B** morning | **Zone C** morning

2 hours

Instructions to students

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

2226 – 5221

11 pages

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 19]

Mathilda works at a local service station that sells lottery scratchcards. Each scratchcard costs $\$1$. The prizes, and the probability of winning each prize according to the lottery company, are shown in the following table.

Prize ($)	0	2	5	10	100	500
Probability	b	0.085	a	0.004	0.002	0.001

(a) Given that the probability of winning $\$5$ is six times the probability of winning $\$100$ for each scratchcard, find the value of

(i) a.

(ii) b. [4]

(b) Determine whether a scratchcard represents a fair game. Justify your answer. [4]

Mathilda wants to determine whether the probability of winning each prize matches the lottery company's claims. She records the prizes from 750 scratchcards. Her data are shown in the following table.

Prize ($)	0	2	5	10	100	500
Number of scratchcards	692	48	7	1	2	0

(c) Write down the probability of winning a $\$2$ prize, according to Mathilda's data. [1]

(This question continues on the following page)

(Question 1 continued)

Mathilda claims that the lottery company has incorrectly stated the probability of winning a $\$2$ prize.

(d) With reference to your answer to part (c), state a reason to

(i) support Mathilda's claim.

(ii) counter Mathilda's claim. [2]

Mathilda decides to perform a χ^2 goodness of fit test using her data at the 10% significance level to test her claim. To prepare for the test and to ensure that her expected frequencies are all greater than 5, Mathilda merges the data for the $\$10$, $\$100$, and $\$500$ prizes into one column, as shown in the following table.

Prize ($)	0	2	5	10, 100, 500
Number of scratchcards	692	48	7	3

(e) (i) Write down the null and the alternative hypothesis for Mathilda's test.

(ii) Find the p-value for the test.

(iii) State the conclusion of Mathilda's test in context. Justify your answer. [6]

(f) State what type of error Mathilda could be making with her conclusion and one way she could reduce the probability of this type of error. [2]

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2. [Maximum mark: 16]

For this question, give all monetary answers to the nearest dollar ($).

When Kelley begins working at Waldron Industries, she opens an investment account. Waldron Industries will match any deposit Kelley makes into her investment account, up to a maximum of 4% of her monthly earnings. For example, if Kelley deposits $\$20$ into her investment account one month, then Waldron Industries will also deposit $\$20$ into her account.

Kelley earns an initial annual salary of $\$60\,000$, which she receives in equal monthly payments.

(a) Find the smallest deposit Kelley needs to make into her investment account each month to maximize the amount that Waldron Industries will match. [2]

Kelley decides to deposit exactly the amount from part (a) at the end of each month and makes the following assumptions:

- the investment account will have a nominal 9.5% annual rate of return, compounded monthly
- her salary will remain constant
- Waldron Industries will continue to match her deposits up to a maximum of 4% of her annual salary
- she will retire after working at Waldron Industries continuously for 40 complete years.

(b) Determine the total amount in Kelley's investment account when she retires, if her assumptions are true. [3]

(c) By considering the validity of two **different** assumptions made by Kelley, state, in context, why the amount in her investment account when she retires may be

(i) significantly **less** than your answer from part (b).

(ii) significantly **more** than your answer from part (b). [2]

After working at Waldron Industries for 10 complete years, Kelley decides to buy a house. She needs to withdraw $\$50\,000$ from her investment account to use as the initial payment on the house.

(d) Calculate how much **less** Kelley will have in her investment account when she retires after withdrawing the $\$50\,000$, given that she continues to deposit the amount from part (a) each month and her assumptions remain true. [6]

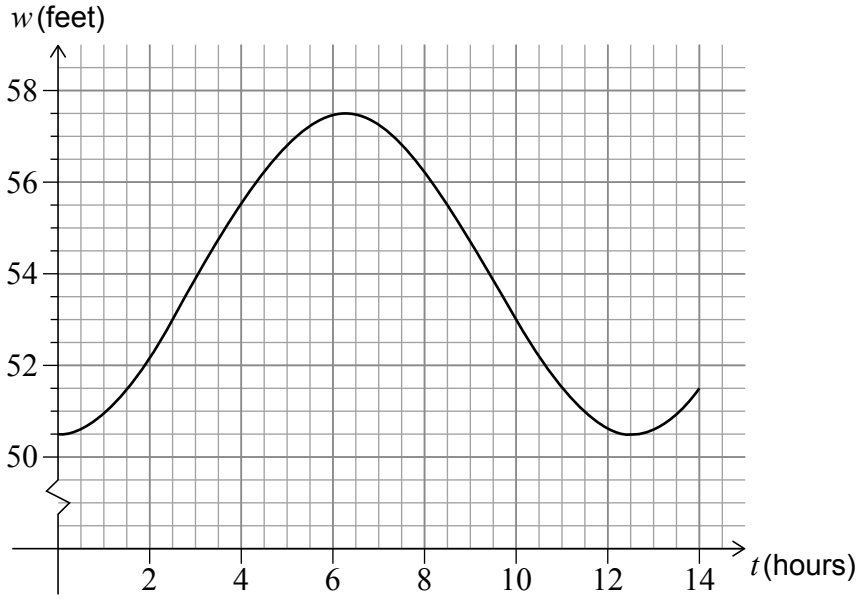
After Kelley withdraws the $\$50\,000$, she wants to increase her monthly deposits into the investment account so that she will still have at least $\$2\,000\,000$ when she retires.

(e) Calculate the minimum necessary increase in Kelley's monthly deposit, given her assumptions remain true. [3]

3. [Maximum mark: 15]

Cargo ships can only enter the Port of Savannah when the water is deep enough. The following graph shows a sinusoidal model of the depth of the water, w feet, on a given day at a time t hours after $00{:}00$ (midnight). The model begins at a minimum of $(0, 50.5)$ and its first maximum is at $(6.25, 57.5)$.

diagram not to scale



(a) Write down the period of the model. [1]

The equation of the model can be expressed in the form

$$w(t) = a\cos(bt) + d \text{ where } b > 0.$$

(b) Determine the equation of the model in this form. [5]

The depth of the water must be at least 53 feet for cargo ships to enter the port.

(c) Find how many hours after midnight $(t = 0)$ the first cargo ship can enter the port on the given day. [2]

(d) Hence, determine the total number of hours between midnight and $12{:}00$ (midday) $(t = 12)$ during which cargo ships can enter the port. [2]

(This question continues on the following page)

(Question 3 continued)

The *Aimmika* is a cargo ship heading for the Port of Savannah. At $t = 0$, the *Aimmika* needs to travel 550 miles to reach the entrance to the Port of Savannah. It travels at an average speed of 21 miles per hour. You can assume that the model from part (b) continues to correctly predict the water depth.

(e) Determine the minimum amount of time the *Aimmika* will need to wait at the entrance before entering the Port of Savannah. [5]

4. [Maximum mark: 13]

When it is $10{:}00$, the minute hand of a clock points to 12 on the clock face and the hour hand of the clock points to 10, as shown in Figure A. Figure B and Figure C show two other positions of the hands as they rotate around the clock face. The minute hand and the hour hand both move smoothly and continuously around the clock face, and it can be assumed that the clock maintains perfect time.

Figure A

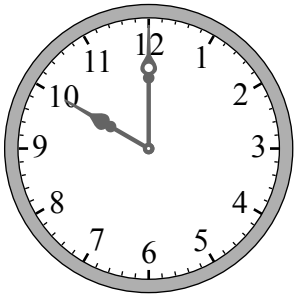


Figure B

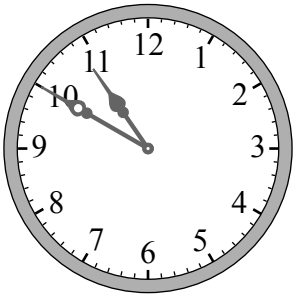
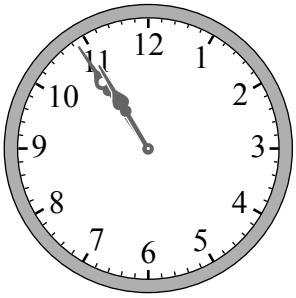


Figure C



In Figure B, the minute hand now points to 10 on the clock face, where the hour hand pointed to in Figure A.

(a) Write down the time taken, in minutes, for the hands to move from their positions in Figure A to their positions in Figure B. [1]

In Figure C, the minute hand now points to the same position that the hour hand pointed to in Figure B.

(b) Find the time taken, in minutes, for the hands to move from their positions in Figure B to their positions in Figure C. Give your answer as a fraction. [2]

(c) Find the time taken, in minutes, for the minute hand in Figure C to move to the position of the hour hand in Figure C. Give your answer as a fraction. [2]

The answers from parts (a), (b) and (c) form a geometric sequence.

(d) Write down the common ratio of this sequence. [1]

The minute hand and the hour hand will be in exactly the same position a seconds after $10{:}54$, where $a \in \mathbb{R}$.

(e) Use your answer from part (d) to find the value of a. [4]

The tip of the minute hand is 11 centimetres from the centre of the clock face.

(f) Find the distance that the tip of the minute hand travels around the clock face between $10{:}00$ and the time found in part (e). [3]

5. [Maximum mark: 15]

A school proposes placing a traffic camera on its road to reduce the number of cars speeding (travelling faster than the speed limit). Saeed is asked to collect a sample of data to determine the number of cars speeding during school hours.

He records the number of cars speeding during each 15-minute interval. His data are shown in the following table.

Number of cars speeding during each 15-minute interval	0	1	2	3	4	5
Frequency (number of 15-minute intervals)	27	47	43	23	20	8

(a) Use the data to estimate the mean and variance of the number of cars speeding during a 15-minute interval. [4]

Saeed suggests that the number of cars speeding in each 15-minute interval may follow a Poisson distribution.

(b) (i) State why your answers to part (a) support Saeed's suggestion.

(ii) State an assumption, in context, that Saeed might make to support using a Poisson distribution as a model. [2]

There are 9 school hours in a school day. Saeed uses $Y \sim \text{Po}(2)$ to model the number of cars speeding during each 15-minute interval during school hours.

(c) According to Saeed's model, find the probability that in one school day

(i) exactly 75 cars are speeding.

(ii) at least 60 cars are speeding. [4]

On the first day the traffic camera is installed, it records 59 cars speeding during school hours.

Saeed assumes the number of cars speeding every 15 minutes still follows the model $Y \sim \text{Po}(\lambda)$. He carries out a hypothesis test at the 5% significance level using the following hypotheses:

$$\text{H}_0: \lambda = 2$$

$$\text{H}_1: \lambda < 2$$

(d) (i) Find the critical region of Saeed's test.

(ii) State, with reason, the conclusion of Saeed's test in context. [5]

6. [Maximum mark: 18]

Two lines, l_1 and l_2, in three-dimensional space have general vector $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ respectively, given by

$$\boldsymbol{r}_1 = \begin{pmatrix} 1 \\ -8 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -3 \\ 2 \end{pmatrix} \text{ and } \boldsymbol{r}_2 = \begin{pmatrix} 8 \\ -5 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -10 \\ 6 \\ -2 \end{pmatrix}.$$

The two lines intersect at point A.

(a) Find the coordinates of point A. [4]

(b) Find the acute angle between l_1 and l_2. [4]

Point B has coordinates $(-17, 10, -2)$.

(c) Show that point B lies on l_2. [3]

Point C is the point on l_1 that is closest to point B.

(d) Find the coordinates of point C. [7]

7. [Maximum mark: 14]

Barefield's Bounty, a commercial fish farm, introduces an initial stock of fish into a pond. After introducing the fish into the pond, Michael, the farm manager, tracks the population growth of the fish. He finds that the population of the fish, $p(t)$, can be modelled using the equation

$$p(t) = \frac{9216}{1 + 6.68\mathrm{e}^{-0.051t}},\ t \geq 0,$$

where t is the number of weeks after the initial stock of fish is introduced into the pond.

(a) Find the number of fish that Barefield's Bounty initially introduced into the pond. [2]

According to Michael's model, the population of the fish will reach a quantity that is double the initial stock of fish after k weeks.

(b) Find the value of k. [2]

(c) Write down the carrying capacity of the pond. [1]

(d) (i) Find $p'(t)$ as a function of t.

(ii) Sketch the graph of $p'(t)$ for $0 \leq t \leq 200$, labelling any axis intercepts.

(iii) With reference to your graph from part (d)(ii), state why $p(t)$ is always increasing. [8]

Michael's model predicts that the population growth of the fish is at a maximum when $t = a$.

(e) Write down the value of a. [1]